METZ, MARX, PANHORST, CLINE.

Investigation of Steel and Invar Tapes

Civil Engineering

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INVESTIGATION

OF

STEEL AND INVAR TAPES

BY

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THESIS

FOR THE

DEGREE OF BACHELOR OF SCIENCE

IN

CIVIL ENGINEERING

COLLEGE OF ENGINEERING

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COLLEGE OF ENGINEERING

June 1, 1915.

I hereby recommend that the thesis prepared under my direction by Carl Altgeld Metz, Frederick August Kuhs Marx, Frederick William Panhorst, and Irl Reuben Cline entitled INVESTIGATION OF STEEL AND INVAR TAPES be accepted as fulfilling this part of the requirements for the degree of Bachelor of Science in Civil Engineering.

R E Davis

Instructor in Civil Engineering.

Recommendation concurred in:

Ira O. Baker.

Head of the Department of
Civil Engineering.

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GENERAL STATEMENT

The purpose of this investigation is to compare results obtained from measuring a base-line with steel and invar tapes, under varying conditions of length of tapes, distance between supports, pull, and temperature. Four different tapes were used; namely, a 100-ft. steel tape, a 100-ft. invar tape, a 300-ft. steel tape, and a 300-ft. invar tape. Pulls of 12, 20, and 30 pounds were used for each tape with different distances between supports. With the 100-ft. tapes, supports were placed at 25, 50, and 100 ft. intervals; and with the 300-ft. tapes, the supports were 25, 50, 75, and 100 ft. apart. The temperature at which measurements were taken varied from 32°F. to 81°F.

In order to maintain a uniform grade for each tape length, two parallel lines of stakes were established along the base-line; one for measurements with the 100-ft. tapes, and the other for measurements with the 300-ft. tapes. In the former, the stakes on which the ends of the tapes were marked, were placed 100 ft. apart; and in the latter, these stakes were placed 300 ft. apart. Hereafter the first mentioned line will be known as the 100-ft. line and the latter as the 300-ft. line.

LOCATION

The base-line is located on the east side of the north and south road, on the east side of the golf links of the University of Illinois, Urbana, Illinois. These golf links lie about

one-half mile south of the campus, just beyond the west division of the Mount Hope Cemetery. The line is not protected in any way from the wind or sun, but lies in the open prairie. The ground surface over which the line is located is gently rolling; thus it was necessary to break grade for each tape length, and in some cases mentioned later, to break grade between the stakes marking the ends of the tapes.

DESCRIPTION OF THE LINES

100-FOOT LINE.

Concrete monuments were set at random about 2400 feet apart to mark the ends of the base-line. Each monument was about 4 ft. long and 7 in. in diameter, and was reinforced with a 4-ft. steel rod, 3/8 in. in diameter. A brass plug 4 in. long and 1/2 in. in diameter was inserted in the top, and protruded slightly above the surface of the concrete. The top of the monument was about 2 in. above the ground surface. The terminals of the line were marked with fine scratches on the brass plugs as shown in Fig. 1.

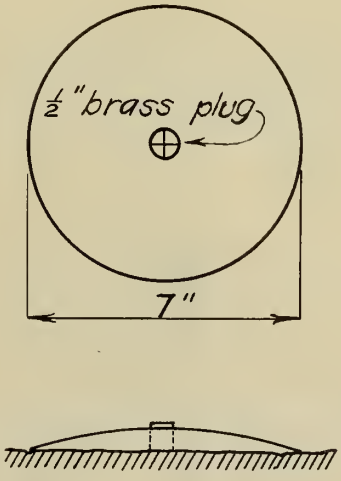

Fig. 1.

Beginning with the monument on the north end of the line, as station 0+00, 2"x4" oak stakes about 4½ ft. long were driven at intervals equal to a tape length. These stakes are shown in Fig. 2, and hereafter will be called station stakes. They were

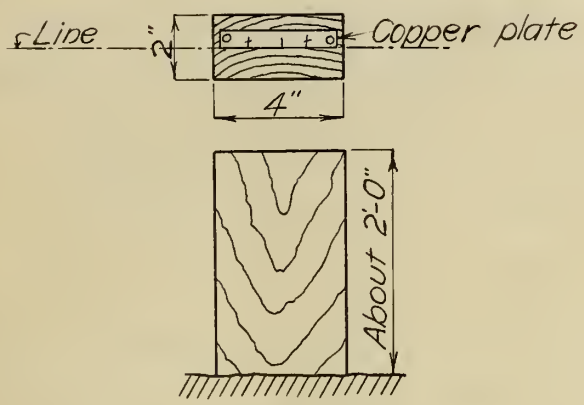

Fig. 2.

driven with their centers on the established line with the 4-in. dimension parallel to the line. After the stakes were firmly driven, the tops were sawed off about 2 ft. above the ground surface. On the top of each stake, a copper strip 4"x 1/2" was tacked, with one edge on line with the terminals.

Tables as shown in Fig. 3 were constructed over each monument. The four uprights, 2"x2", were driven firmly into the

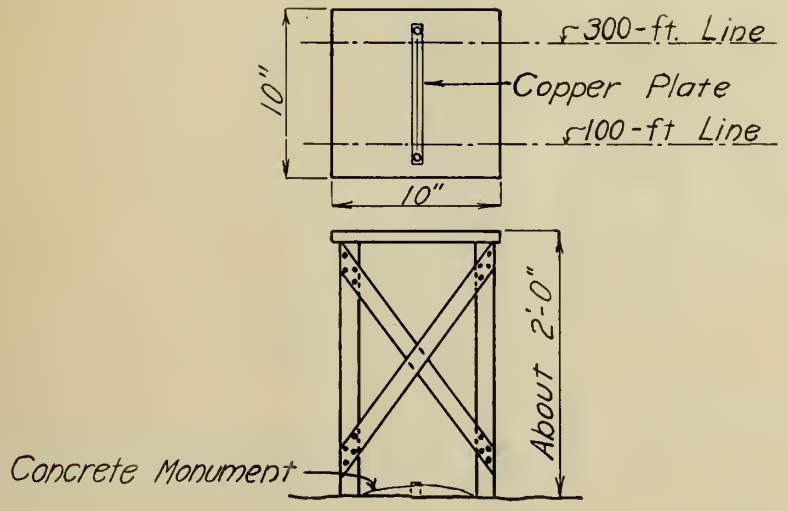

Fig. 3.

ground close to the monument, and the tops were sawed off at the same elevation, about 2 ft. above the ground surface. A 1-in. board about 10 in. square was nailed on the top of the uprights, which were securely cross braced. To transfer the point at each end from the monument to the top of the table, a transit was set up over the terminal of the line before the top of the table was placed, and a line was established at right angles to the base-line. By setting up the transit over a point on the line just located, at a distance of about 40 ft. from the monument, and sighting on the terminal as marked on the brass plug, the line of sight was transferred to a copper strip nailed to the top of the table. The transit was then set up approximately on line with the station stakes, and the line of sight to the monument was again projected to the top of the table. The intersection of this line with the line of sight at right angles, marked the exact terminal of the base-line on this table.

Wooden support stakes 1"x1" and 4 ft. long were driven at the 25-ft. points with one edge on line with the centers of the station stakes. A sketch of a support stake is shown in Fig. 4.

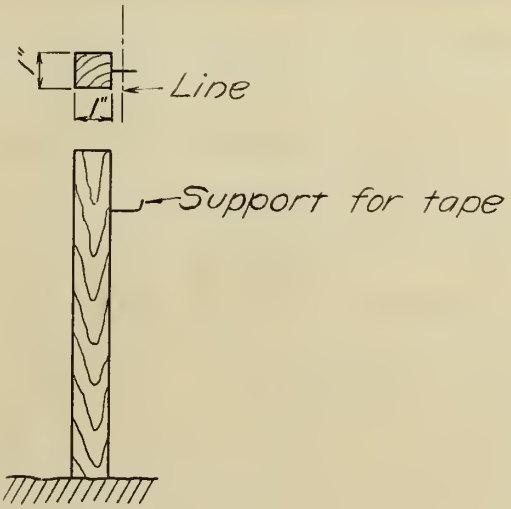

Fig. 4.

When necessary to preserve alignment, the support stakes were braced with laths. The supports for the tape were furnished by square screw hooks 1½ in. long, set in the support stakes on grade with the tops of the two adjacent station stakes. The elevation for the hooks on the intermediate stakes was established by "shooting in" the grade.

300-FOOT LINE.

The terminals of the 300-ft. line were established by measuring a distance of approximately 6 in. to one side of the 100-ft. line, on the terminal lines previously established on the copper strips at right angles to the base-line. Station stakes and intermediate stakes for the 300-ft. line were placed in the same way as those for the 100-ft. line, except that the station stakes were 300 ft. apart instead of 100 ft. as shown on the profile, Plate V, page 11, A 7-ft. knoll near the middle of the base-line necessitated breaking the grade at a point midway between station stakes. When using 75-ft. and 100-ft. supports with pulls of 12 and 20 lbs., the grade was broken between station stakes in a few cases to prevent the tape from dragging on the ground. These changes in the grade line are discussed more fully on page 24.

PRELIMINARY PROCEDURE

After the lines had been established, a line of differential levels were run from the United States Geological Survey bench mark at the southeast corner of Engineering Hall, to each of the monuments. The object of these levels, which are tabulated on Plate I, page 6, was to tie the base-line to sea level elevations, so that a correction could be made to obtain the corres-

PLATE I.

DIFFERENTIAL LEVELS

Sta.	B.S.	H.I.	F.S.	Elev.	
B.M.	5.55	726.60		721.05	B.M. east of Eng. Hall.
⊙	4.78	739.92	1.46	725.14	
⊙	9.08	733.59	5.41	724.51	
⊙	13.09	744.31	2.37	731.22	
⊙	9.91	752.80	1.42	742.89	
⊙	5.31	757.80	0.31	753.49	
⊙	7.14	763.45	1.49	756.31	
⊙	8.36	771.00	0.81	762.64	
⊙	5.45	765.93	10.52	760.48	Top of N.W. post of table on N. end of line.
B.M.			0.91	765.02	S.E. corner of Levi Kisby's tomb stone.

ponding length at sea level if desired; but this correction was not made in this investigation, however, since the object was merely a comparison of measurements.

Profile levels were run over the ground, and over the tops of the station stakes on both lines. The notes for the profile of the ground surface are tabulated in Plate II, page 8; while those for the grades on the 100-ft. and 300-ft. lines are given in Plates III and IV respectively, pages 9 and 10. Plate V, page 11, shows a profile of the ground surface and the established grades on the 300-ft. and 100-ft. lines respectively.

After the copper strips had been placed upon the station stakes, and the support stakes had been properly aligned, with the screw hooks at the desired elevation, the line was ready for the measurements.

EQUIPMENT

The equipment of the party for measurements taken during the day consisted of 4 tapes, 2 spring balances, 2 Fahrenheit thermometers, 2 4-ft. staffs for tape stretchers, a marker, and a note-book. The equipment used for taking measurements at night was the same as that mentioned above with the addition of four electric flash lights.

TAPES.

All of the tapes had been previously standardized at the United States Bureau of Standards at Washington, D.C. The correct length for each tape, at 62°F., was determined when subjected to pulls of 12, 20, and 30 lbs., the invar tapes being supported at

PLATE II.

PROFILE LEVELS OF GROUND

Sta.	B.S.	H.I.	F.S.	Elev.
24+12	7.3	745.8		738.5
23+50			5.9	739.9
23+00			5.4	740.4
21+00			5.0	740.8
20+00			3.6	742.2
19+00			1.6	744.2
0	7.8	753.0	0.8	745.2
18+00			7.2	745.8
17+00			5.7	747.3
16+00			5.4	747.6
15+00			4.1	748.9
14+00			1.7	751.3
13+00			1.3	751.7
0	5.2	758.1	0.1	752.9
12+00			5.9	752.2
11+00			5.8	752.3
10+00			5.0	753.1
9+00			2.4	755.7
0	3.4	760.4	1.1	757.0
8+00			1.5	758.9
7+60			0.8	759.6
7+00			2.2	758.2
6+00			6.2	754.2
5+00			6.7	753.7
4+50			6.2	754.2
4+00			5.0	755.4
0	2.8	758.5	4.7	755.7
3+00			3.2	755.3
2+40			3.7	754.8
2+00			5.0	753.5
1+00			3.5	755.0
0	4.4	761.2	1.7	756.8
0+00			2.4	758.8

PLATE III.

LEVELS OVER 100-FT. STATIONS

Sta	B.S.	H.I.	F.S.	Elev.
0+00	0.60	761.18		760.58
1+00			4.18	757.00
2+00			5.75	755.43
3+00			3.99	757.19
4+00			3.86	757.32
5+00			3.51	757.67
6+00	6.51	762.59	5.10	756.08
7+00			2.49	760.10
8+00			1.75	760.84
9+00			5.69	756.90
10+00			8.07	754.52
11+00			8.92	753.67
12+00	0.95	754.65	8.89	753.70
13+00			0.95	753.70
14+00			1.44	753.21
15+00			3.70	750.95
16+00			4.86	749.79
17+00			5.18	749.47
18+00	0.34	748.36	6.63	748.02
19+00			2.07	746.29
20+00			3.90	744.46
21+00	3.28	746.11	6.53	742.83
22+00			3.56	742.55
23+00			3.53	742.58
24+00			5.17	740.94
24+12			5.59	740.52
B.M.			8.22	737.89

B.M.= N.W. corner of east gate post.

PLATE IV.

LEVELS OVER 300-FT. STATIONS

Sta.	B.S.	H.I.	F.S.	Elev.	
B.M.	5.37	766.65		761.28	
0+00			6.12	760.53	
3+00	6.08	763.33	9.40	757.25	
6+00			7.25	756.08	
7+50			1.94	761.39	
9+00	0.02	756.98	6.37	756.96	
12+00			3.42	753.56	
15+00	0.70	752.52	5.16	751.82	
18+00			4.60	747.92	
21+00	3.89	745.63	9.77	742.75	
24+00			4.81	740.82	
24+12			5.11	740.52	Top of table S end of line.

PROFILES

PLATE V.

LINE FOR 300 FT. TAPES.

Grade B

Sta. 0+00
Elev. 760.53

Sta. 3+00
Elev. 757.25

Sta. 6+00
Elev. 756.08

Sta. 7+50
Elev. 761.39

Sta. 9+00
Elev. 756.96

Sta. 12+00
Elev. 753.56

Sta. 15+00
Elev. 751.82

Sta. 18+00
Elev 747.92

Sta. 21+00
Elev. 742.75

Sta. 24+00 Elev. 740.82
Sta. 24+12 Elev 740.52

Elevation of ground surface

LINE FOR 100 FT. TAPES

Grade A

Sta. 0+00
Elev. 760.53

Sta. 1+00. Elev. 757.00

Sta. 2+00. Elev. 755.43

Sta. 3+00. Elev. 757.19

Sta. 4+00. Elev. 757.32

Sta. 5+00. Elev. 755.67

Sta. 6+00. Elev. 756.08

Sta. 7+00. Elev. 760.10

Sta. 8+00. Elev. 760.84

Sta. 9+00. Elev. 756.90

Sta. 10+00. Elev. 754.52

Sta. 11+00. Elev. 753.67

Sta. 12+00. Elev. 753.70

Sta. 13+00. Elev. 753.70

Sta. 14+00. Elev. 753.21

Sta. 15+00. Elev. 750.95

Sta. 16+00. Elev. 749.79

Sta. 17+00. Elev. 749.47

Sta. 18+00. Elev. 748.02

Sta. 19+00. Elev. 746.29

Sta. 20+00. Elev. 744.46

Sta. 21+00. Elev. 742.83

Sta. 22+00. Elev. 742.55

Sta. 23+00. Elev. 742.58

Sta. 24+00. El. 740.94
Sta. 24+12. El. 740.52

Elevation of ground surface

SCALE.
Vertical. 1" = 10'
Horizontal. 1" = 400'

100-ft. intervals and the steel tapes being standardized flat. The coefficient of expansion, per 1 degree Fahrenheit, of the 100-ft. invar tape was found to be 0.0000004 over a range of from 62° to 88°F.; the 300-ft. invar tape, 0.00000035 over a range of from 31° to 85°F.; the 100-ft. steel tape, 0.0000062; and the 300-ft. steel, 0.00000646. Copies of the standardization certificates are shown in Plates VI to IX inclusive, pages 13 to 16.

BALANCES.

The spring balances were of the dial type, graduated to 1/20 lb. They were used as shown in Fig. 5, to determine the pull on the tape.

Fig. 5.

Each balance was calibrated in a horizontal position, by suspending standard weights in a pan, and noting the error when the weight was applied to the balance. The correction was made by adjusting a movable pointer on the dial of the balances so that the dial readings would be correct. These corrections are tabulated in Plate X, page 17.

PLATE VI.

This certifies that 100-foot INVAR Tape No. 2459 has been compared with the standards of the United States and for the intervals indicated, has the following corrections at 62° Fahrenheit under the following conditions.

Supported at 0- and 100-foot points.

Tension: 12 lb

Interval	Correction
(0 to 100 ft.)	-0.099 foot.

Tension: 20 lb.

(0 to 100 ft.)	-0.030 foot.

Tension: 30 lb.

(0 to 100 ft.)	-0.004 foot.

NOTE: The comparisons of this tape were made at a temperature of 88°F. and in reducing to 62°F. the coefficient of expansion of the tape is assumed to be 0.0000004.

PLATE VII.

This certifies that 100-foot STEEL Tape No. 2432 has been compared with the standards of the United States and for the intervals indicated, has the following corrections at 62° Fahrenheit when supported horizontally throughout its entire length

Interval	Correction
Tension: 12 lb.	
(0 to 100 ft.)	-0.0064 foot.
Tension: 20 lb.	
(0 to 100 ft.)	+0.0015 foot.
Tension: 30 lb.	
(0 to 100 ft.)	+0.0109 foot.

NOTE: The foregoing comparisons were made at the fine lines etched by the Bureau about one inch to one side of the 0- and 100-foot sleeves. They have an uncertainty of 3 in the last place.

The coefficient of expansion of this tape, 0.0000062 per degree Fahrenheit is uncertain by 1 in the last place.

PLATE VIII.

This certifies that the 300-foot STEEL Tape No. 2433 has been compared with the standards of the United States and, for the intervals indicated, has the following corrections at 62°F. when supported horizontally throughout its entire length.

Interval	Correction
Tension: 12 lb.	
(0 to 100 ft.)	-0.0054 foot.
(0 to 200 ft.)	-0.0108 "
(0 to 300 ft.)	-0.0410 "
Tension: 20 lb.	
(0 to 100 ft.)	+0.0005 foot.
(0 to 200 ft.)	+0.0013 "
(0 to 300 ft.)	-0.0330 "
Tension: 30 lb.	
(0 to 100 ft.)	+0.0079 foot.
(0 to 200 ft.)	+0.0158 "
(0 to 300 ft.)	-0.0013 "

NOTE: The foregoing corrections were made at the fine lines etched by this Bureau about two inches to the left of the 0-, 100-, 200-, and 300-foot sleeves. Their uncertainty ranges from 3 in the last place for the small intervals to about 5 for the total length.

The coefficient of expansion for this tape is 0.00000646.

PLATE IX.

This certifies that the 300-foot INVAR Tape No. 2460 has been compared with the United States standards and, for the intervals indicated, has the following corrections at 62°F. under the following conditions.

Supported at 0-, 100-, 200-, and 300 foot points.

Interval	Correction
Tension: 12 lb.	
(0 to 100 ft.)	-0.090 foot.
(0 to 200 ft.)	-0.182 "
(0 to 300 ft.)	-0.274 "
Tension: 20 lb.	
(0 to 100 ft.)	-0.031 foot.
(0 to 200 ft.)	-0.063 "
(0 to 300 ft.)	-0.095 "
Tension: 30 lb.	
(0 to 100 ft.)	-0.006 foot.
(0 to 200 ft.)	-0.013 "
(0 to 300 ft.)	-0.020 "

NOTE: The coefficient of expansion of this tape as determined over the range from 31° to 85° is 0.00000035.

PLATE X.

WEIGHT OF TAPES

Tape	Net Weight lb.	Net Weight lb. per ft.
300 I	5.4	0.0180
300 S	5.0	0.0166
100 I	1.8	0.0180
100 S	1.3	0.0130

CALIBRATION OF SPRING BALANCES

Horizontal Position

	Zero Reading lb.
Balance No. 1	-1.5
Balance No. 2	-1.9

Vertical Position

Load Applied lb.	Balance No.1 Reading lb.	Balance No.2 Reading lb.
0	0.0	0.0
10	10.0	10.0
20	19.7	19.8
30	29.7	29.7

CALIBRATION OF THERMOMETERS

Comparative readings- degrees Fahrenheit

Thermometer No.	1	2	3	4	Standard
In solution of ice and water	32.0	32.0	32.0	32.0	32.0
Room temperature	67.8	67.8	67.8	68.0	68.5

Four Fahrenheit thermometers were calibrated by comparing them with standard thermometers at 32°F. and 68°F. Notes for these comparisons are shown in Plate X, page 17. Two of these thermometers were held in reserve, as only two were used at the same time in the field, one at the head end of the tape, and one at the rear end. The purpose of the arrangement of the thermometers was to secure a more accurate average of the temperature during each measurement than could have been attained with one thermometer or with two at the same place, there being a possibility of a slight difference in temperature at the two ends of the tapes, as shown in the field notes Plate XI, page 23. The weight of each tape was determined for use in computing the corrections which will be described in detail later. The weights of the tapes are shown in Plate X, page 17.

TAPE STRETCHERS.

As shown in Fig. 5, the 4-ft. staffs were used to insure a steady pull on the tape while the marking of the ends of the tapes was in progress. The principle of leverage involved, and the comfortable position of the chainmen in holding the tape are clearly shown in the figure.

ORGANIZATION OF FIELD PARTY

Four men comprised the party in the field. One man, acting as recorder, also marked the forward end of the tape on the copper strips at each station stake, and read the thermometer at the head end of the tape. A head chainman and a rear chainman observed that the tapes were properly supported, and that the balances on their respective ends of the tape were registering the desired pull preparatory to the marking of the tape ends.

As the head chainman went forward, he placed the tape on the proper support stakes. The fourth man's duties as observer were to hold the rear end of the tape on the scratch previously made by the recorder, and to read the thermometer at the rear end of the tape.

METHOD OF MEASUREMENT

The method of measuring was the same for both lines. The measurement of the 300-ft. line, for example, proceeded as follows: The tape was carefully undone, so as not to form any bends which if present would have changed the length of the tape, since the 12 lb. pull was not sufficient to stretch the tape very tightly. Beginning at the north end of the line, station 0+00, the head chainman proceeded to station 3+00, placing the tape on the supports at the desired interval, on his way. A thermometer was hung on a wire fence along side of the line, at each end of the tape, at about the same height above the ground as the tape. The two chainmen hooked their balances into the rings at the ends of the tape, and held the balances horizontally at such a height that the tape would be horizontal and barely touching the station stake. In this position there was no unnecessary friction between the parts of the balance, or between the station stake and the tape due to the bending of the tape around one edge of the stake.

When the required pull was obtained, and each chainman was sure that the pull would remain constant, the observer, upon a signal from the rear chainman held the zero mark of the tape on the terminal of the 300-ft. line. The ends of the tape were placed at the sides of the copper strips, which were about the same thickness as the tape. In this way the scratch could be made more accurately than if the tape had been held on top of the strip, and

also, the rear end of the tape could be more accurately held on the mark. When the zero mark of the tape coincided with the initial mark on the table, the observer flipped the tape to set up vibrations in order to equalize the tension throughout the tape length, and after vibration had ceased he signalled to the recorder, who made a scratch on the copper strip at station 3+00, thus marking off a 300-ft. length. The scratch was made by a marker, illustrated in Fig. 6, which consisted of a needle inserted in a wooden handle. The two temperature readings were then taken and recorded in the note book, together with the numerical value of the measured distance between the initial point on the line, and the scratch on the station stake at station 3+00.

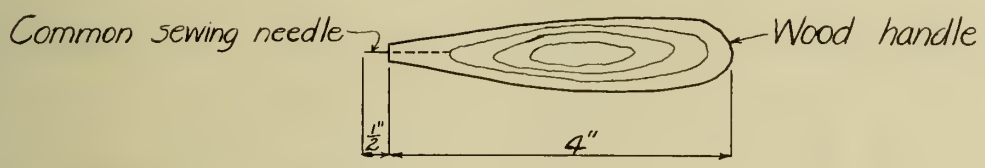

Fig. 6.

Before measurements were taken, it was thought that there would be need of replacing the copper strips from time to time as scratches became more numerous; however, it was not necessary to replace the plates, since the bright copper exposed by the fine scratches oxidized during the intervals between field periods. Since from 6 to 16 measurements were taken in succession at each period, it was necessary to distinguish the proper mark for the terminal of the tape, from those marks which were used in a previous measurement. This distinction was brought about by using a single scratch for the desired mark, and crossing out all marks made previously. This is more clearly shown on the copper strip shown in Fig. 2, page 3. In some cases, as will be noted in the

measurement between stations 9+00 and 12+00, in measurement No. 27, page 23, owing to the forward end of the tape not falling on the copper strip when the tape was in position for measurement, it was necessary to add to, or subtract from, the measured tape length, a fractional part of a foot, so that a mark could be made on the copper strip at the forward end. The addition or subtraction was made in tenths of a foot, and was accomplished by using a 6-inch celluloid scale graduated to 1/50 in. In a similar manner this scale was used at the ends of the base-line to determine the distance to the nearest 1/10,000 ft.

After the first tape length was marked off and recorded, the head chainman proceeded to station 6+00, placing the tape on the desired support stakes as before, and the cycle of procedure was repeated. The measuring proceeded in this manner for each station stake, until station 24+00 was reached. At this point it was necessary to use a fractional part of the tape length, as the southern terminal of the base-line was only about 12 ft. beyond station 24+00. Thus the head chainman advanced to a point just beyond the end table and applied a tension to the tape in the same manner as at a station stake. With the rear chainman on line with the stakes, and located at about station 21+12, the tape was subjected to the required pull, and the distance to the southern terminal of the base-line was noted. On the return trip to the north end of the line, the measuring proceeded in exactly the same manner.

EXPLANATION OF FIELD NOTES

Plate XI, page 23, is a typical page from the note-book. At the top of the page is the number of the measurement,

which facilitates reference to subsequent tables and computations. Immediately following the measurement number is a set of brief notes giving the length of tape, kind of tape, pull to which the tape was subjected, and the distance between supports. In the upper right hand corner of the page is the date and condition of the weather.

Under the five column headings are given the location of the rear and head end of the tape for each tape length, the temperature observed at each end of the tape as described in detail before on page 18, and the measured length between successive station stakes.

The average of the observed temperature was found and inserted at the foot of columns 3 and 4. The total measured length of the line was obtained by adding the partial lengths in column 5, and was inserted at the bottom of the column. Later, after the corrections for sag, grade, temperature, and length were computed for each measurement, the values of these corrections were inserted at the bottom of the page. The total correction was then computed from the four corrections just mentioned, and applied to the measured length to obtain the value of the corrected length recorded at the bottom of the page.

PLATE XI.

PAGE FROM FIELD BOOK

Measurement No. 27 March 13, 1913

Cloudy

300-ft. Steel 30# pull 25 ft. support Damp

Sta.	to Sta.	Temp. F.	Temp. F.	Measured Length
N	3+00	32.5	32.0	300
3+00	6+00	32.2	32.6	300
6+00	9+00	31.8	32.5	300
9+00	12+00	32.2	32.8	300.1
12+00	15+00	32.3	33.0	300
15+00	18+00	32.5	33.0	300
18+00	21+00	34.0	33.8	300
21+00	24+00	34.0	33.3	300
24+00	S	32.2	32.2	12.6900
		293.7	295.2	2412.7900
	Average	32.63	32.80	
	Average	32.71		

Corrections

Sag	-0.0194
Grade	-0.2836
Temp.	-0.4563
Length	-0.0104
	-0.7697

Corrected length 2412.0303

CORRECTIONS

For each measurement, corrections for grade, sag, temperature, and length of tape were computed as shown in Plate XIV, page 31. It was unnecessary to compute corrections for pull, since the tapes had been standardized for each of the tensions used in the field measurements. The corrections for the equivalent lengths at sea level were also omitted for reasons already explained on page 5.

Owing to the 300-ft. tapes dragging on the ground between supports when subjected to 12 and 20 lb. pulls with 75 and 100-ft. intervals between supports, it was necessary to establish four different grade lines for the measurements of the base-line. The four grades referred to were as follows: Grade A as shown in the notes in Plate III, page 9, and in profile in Plate V, page 11, was used for all measurements on the 100-ft. line, and had all support stakes set on a uniform grade between successive station stakes. Grade B also had uniform grades between station stakes except between stations 6+00 and 9+00. In this interval, the grade was uniform between stations 6+00 and 7+50, and between stations 7+50 and 9+00, as shown on the profile. This grade line, B, was used for all measurements on the 300-ft. line except as noted in Plate XIV, pages 31 to 34. In a few cases, as mentioned before, grades C and D were used on the 300-ft. line to prevent the tapes from dragging on the ground. Both C and D are the same as grade B with the following corrections applied to the supports at the stations listed below.

Grade C:

Station 8+25 raised 0.5 ft.

Station 13+50 raised 0.5 ft.

Station 23+25 raised 0.5 ft.

Grade D:

Station 8+00 raised 2.0 ft.

Station 13+00 raised 1.0 ft.

Station 23+00 raised 1.0 ft.

A sample sheet of all of the corrections for each measurement is shown in Plate XII, page 27. In this particular instance the corrections apply to measurement No. 27, which has been already discussed, and shown in the note-book form in Plate XI, page 25.

GRADE CORRECTION:

The formulas from which the various corrections were computed are those used by the United States Coast and Geodetic Survey on similar work. The grade correction was determined from the formula $C_g = -\frac{\Sigma h^2}{2l}$, where C_g is the correction in feet for the entire line, l is the distance in feet between the breaks in the grade, and h is the difference in elevation in feet between successive breaks in the grade. As will be noted from a study of Plate XII, the computations were greatly simplified by arranging the work in tabular form. The figures in the column headed "h" are the differences in elevation of adjacent station stakes, while those in the column headed "h_1" and "h_2" are the differences in elevation of adjacent points 150 and 12.5 ft. apart, respectively. The subdivision of the differences in elevations into these three columns was necessary, since the correction formula applies only

when the breaks in grade fall at equal intervals. Therefore, by summing up the three partial corrections as shown at the bottom of the plate, the total grade correction C_g is obtained.

SAG CORRECTION.

The formula for the sag correction in feet is $C_s = -\frac{L}{24}\left(\frac{wd}{p}\right)^2$, where L is the measured length of the base-line in feet, w is the weight of the tape per foot of length, d is the distance between supports in feet, and p is the tension applied to the tape in pounds. The values substituted for the various symbols were taken from Plate XIV, page 31, and Plate X, page 17.

TEMPERATURE CORRECTION.

The temperature correction was computed from the formula $C_t = +\alpha L(T_m - T_o)$, where C_t is the total correction to be applied to the base-line in feet, α is the coefficient of expansion for the tape, L is the measured length of the line in feet, T_m is the average temperature of the period during which the line was measured, and T_o is the temperature at which the tape was standardized. The values of α and T_o are obtained from the standardization certificates on pages 13 to 16. L and T_m are obtained from Plate XIV, pages 31 to 34.

LENGTH CORRECTION.

The correction for length is determined from the formula $C_l = B\frac{L}{l}$, where C_l is the total length correction in feet; B is the correction in feet per tape length as shown in the standardization certificates, Plates VI to IX; L, as before, is the measured length of the line; and l is the length of the tape in feet.

PLATE XII.

CORRECTIONS FOR MEASUREMENT NO. 27

Grade Correction:

$$C_G = -\frac{\Sigma h^2}{2 l}$$

C_G = correction for grade.

l = distance between breaks in grade

h = difference in elevation between adjacent stations.

Sta.	Elev. Ft.	h	h^2	h_1	h_1^2	h_2	h_2^2
0+00	700.53						
3+00	57.25	3.28	10.7584				
6+00	56.08	1.17	1.3689				
7+50	61.39			5.31	28.1961		
9+00	56.96			4.43	19.6249		
12+00	53.56	3.40	11.5600				
15+00	51.82	1.74	3.0276				
18+00	47.92	3.90	15.2100				
21+00	42.75	5.17	26.7289				
24+00	40.82	1.93	3.7249				
24+12.5	40.52					0.30	0.0900
		Σh^2 =	72.3787	Σh_1^2 =	47.8210	Σh_2^2 =	0.0900
		l =	300	l_1 =	150	l_2 =	12.5
		$-\frac{\Sigma h^2}{2 l}$ =	-0.12063	$-\frac{\Sigma h_1^2}{2 l_1}$ =	-0.15940	$-\frac{\Sigma h_2^2}{2 l_2}$ =	-0.0036
			-0.15940				
			-0.00360				
		C_G =	-0.28360				

Sag Correction: $C_s = -\frac{L}{24}\left(\frac{wd}{r}\right)^2$

C_s = correction for sag.

L = length of base line or 2412.5 ft.

w = weight of tape per ft., or $\frac{5}{300}$ lb.

d = distance between supports, or 25 ft.

r = pull in pounds, or 30 lb.

$$C_s = -\frac{2412.5}{24}\left(\frac{5 \times 25}{300 \times 30}\right)^2 = -0.0194 \text{ ft.}$$

Temperature Correction: $C_t = -aL(T_m - T_o)$

a = coefficient of expansion for tape 0.00000646

L = length of base line or 2412.5 ft.

T_m = average temperature during observation 32.71°

T_o = temperature at which tape was standardized or 68.0°

$$C_t = +0.00000646 \times 2412.5(32.71-68.0) = -0.4883 \text{ ft.}$$

Length Correction: $C_l = B\frac{L}{l}$

B = correction per tape length or -0.0013

L = length of base line or 2412.5 ft.

l = length of tape or 300 ft.

$$C_l = -0.0013 \times \frac{2412.5}{300} = -0.01045 \text{ ft}$$

Sea Level Correction:

Since these measurements were made for comparative purposes, the correction to sea level was unnecessary, as it is constant throughout.

PLATE XIII.

SAG CORRECTIONS

Tape	Pull lb.	Distance between supports - ft. 25	50	75	100
100 S	12	-0.0734	-0.2934		-1.1737
100 S	20	-0.0265	-0.1057		-0.4225
100 S	30	-0.0118	-0.0470		-0.1877
300 S	12	-0.1207	-0.4825	-1.0856	-1.9298
300 S	20	-0.0434	-0.1737	-0.3907	-0.6946
300 S	30	-0.0193	-0.0772	-0.1736	-0.3087
100 I	12	-2.1103	-1.6874	-0.9841	- 0
and	20	-0.7592	-0.6074	-0.3544	0
300 I	30	-0.3375	-0.2700	-0.1575	0

LENGTH CORRECTIONS

Tape	Pull in pounds 12	20	30
100 S	-0.1544	-0.0362	-0.2629
100 I	-2.3879	-0.7236	-0.0965
300 S	-0.3296	-0.1849	-0.0105
300 I	-2.2030	-0.7638	-0.1608

GRADE CORRECTIONS

Grade	Dist. between station stakes	Correction
A	100	-0.4100
B	300	-0.2836
C	300	-0.2935
D	300	-0.3436

To facilitate the application of the corrections, the results of the computations were tabulated in the form shown in Plate XIII. Since the temperature correction alone was an independent value for each measurement, the corrections for sag, grade, and length could be readily taken from the table.

Plate XIV gives a summary of data and results for the entire series of measurements in the order in which the measurements were taken. No definite system was followed out in the order of taking measurements, as will be seen from the tables. The letters in the column with the heading "Grade", refer to the grade lines discussed on page 24, which were used for the measurement in question. Observation numbers followed by the letter N indicate that the measurement was taken at night.

GROUPING OF MEASUREMENTS

In order to compare the corrected distances of the various measurements more readily, all measurements were grouped with reference to the tape length, kind of tape, distance between supports, and pull, as shown in Plates XV and XVI, pages 35 to 42. In plate XV the various groups are combined with reference to the distance between supports, while in Plate XVI the classification is with respect to the length and kind of tape. In both Plates, are also found the averages of the smaller groupings, which were used in the determination of the probable error. These averages are discussed more fully in the explanation regarding the computations of the probable errors on page 43.

PLATE XIV. Data and Results of Observations.

Observation No.	Length of Tape feet	Kind of Tape	Dist. between Supports	Pull lbs.	Grade Line	Average Temperature °Fahr.	Measured Distance feet	Sag Correction feet	Grade Correction feet	Temperature Correction feet	Length Correction feet	Corrected Distance feet	Observation No.
1	300	Steel	25'	12	B	48.26	2412.3383	-0.1208	-0.2836	-0.2141	-0.3296	2411.3902	1
2	"	"	"	"	"	47.81	12.7417	"	"	-0.2211	"	11.7866	2
3	"	"	"	20	"	48.01	12.4933	-0.0435	"	-0.1997	-0.1849	11.7816	3
4	"	"	"	"	"	49.74	12.4633	"	"	-0.1912	"	11.7601	4
5	"	Invar	"	12	"	52.28	12.5277	+2.1103	"	-0.0078	-2.2030	12.1436	5
6	"	"	"	"	"	51.88	12.5343	"	"	-0.0085	"	12.1495	6
7	"	"	"	20	"	53.15	12.2645	+0.7592	"	-0.0075	-0.7638	11.9688	7
8	"	"	"	"	"	53.77	12.2905	"	"	-0.0069	"	11.9954	8
9	"	"	"	30	"	52.09	12.0067	+0.3375	"	-0.0084	-0.1608	11.8914	9
10	"	"	"	"	"	51.12	12.0083	"	"	-0.0091	"	11.8923	10
11	"	Steel	"	"	"	49.37	12.5783	-0.0193	"	-0.1968	-0.0105	12.0681	11
12	"	"	"	"	"	49.78	12.3450	"	"	-0.1904	"	11.8412	12
13	100	"	"	12	A	46.75	12.5842	-0.0734	-0.4100	-0.2280	-0.1545	11.7183	13
14	"	"	"	"	"	48.72	12.6833	"	"	-0.1986	"	11.8468	14
15	"	"	"	20	"	50.07	12.2900	-0.0265	"	-0.1784	+0.0363	11.7114	15
16	"	"	"	"	"	52.21	12.3750	"	"	-0.1464	"	11.8284	16
17	"	"	"	30	"	53.68	11.9982	-0.0118	"	-0.1244	+0.2628	11.7148	17
18	"	"	"	"	"	53.74	12.1033	"	"	-0.1235	"	11.8208	18
19	"	Invar	"	12	"	55.73	12.6250	+2.1103	"	-0.0060	-2.3879	11.9314	19
20	"	"	"	"	"	54.32	12.6517	"	"	-0.0074	"	11.9567	20
21	"	"	"	20	"	52.40	12.3700	+0.7592	"	-0.0093	-0.7236	11.9863	21
22	"	"	"	"	"	53.47	12.4192	"	"	-0.0082	"	12.0366	22
23	"	"	"	30	"	51.71	12.0942	+0.3375	"	-0.0099	-0.0965	11.9153	23
24	"	"	"	"	"	50.38	12.2342	"	"	-0.0112	"	12.0540	24
25	300	Steel	"	12	B	32.13	13.1792	-0.1207	-0.2836	-0.4654	-0.3296	11.8799	25
26	"	"	"	20	"	32.21	12.8067	-0.0434	"	-0.4647	-0.1849	11.8307	26

PLATE XIV Con't

Observation No.	Length of Tape ft.	Kind of Tape	Dist. between Supports	Pull lbs.	Grade Line	Average Temperature °Fahr.	Measured Distance ft.	Sag Correction ft.	Grade Correction ft.	Temperature Correction ft	Length Correction ft	Corrected Distance ft.	Observation No
27	300	Steel	25	30	B	32.71	2412.7900	-0.0194	-0.2836	-0.4563	-0.0104	2412.0203	27
28	"	"	"	12	"	34.35	12.9592	-0.1207	"	-0.4308	-0.3296	11.7945	28
29	"	Invar	"	"	"	35.35	12.5500	+2.1103	"	-0.0225	-2.2030	12.1512	29
30	"	"	"	20	"	33.87	12.2750	+0.7592	"	-0.0237	-0.7638	11.9631	30
31	"	"	"	30	"	33.65	12.0117	+0.3375	"	-0.0239	-0.1608	11.8809	31
32	"	"	"	12	"	34.71	12.5600	+2.1103	"	-0.0230	-2.2030	12.1607	32
33	100	Steel	"	"	A	35.59	12.8717	-0.0734	-0.4100	-0.3948	-0.1544	11.8391	33
34	"	"	"	20	"	35.79	12.6283	-0.0265	"	-0.3918	+0.0362	11.8362	34
35	"	"	"	30	"	34.39	12.4333	-0.0118	"	-0.4128	+0.2629	11.8616	35
36	"	"	"	12	"	36.12	12.8633	-0.0734	"	-0.3869	-0.1544	11.8386	36
37	"	Invar	"	"	"	34.38	12.6617	+2.1103	"	-0.0265	-2.3879	11.9476	37
38	"	"	"	20	"	33.95	12.4042	+0.7592	"	-0.0269	-0.7236	12.0029	38
39	"	"	"	30	"	33.54	12.1267	+0.3375	"	-0.0273	-0.0965	11.9304	39
40	"	"	"	12	"	32.27	12.6767	+2.1103	"	-0.0285	-2.3879	11.9606	40
41	300	Steel	50	"	B	33.91	13.2500	-0.4825	-0.2836	-0.4376	-0.3296	11.7167	41
42	"	"	"	"	"	34.42	13.2750	"	"	-0.4297	"	11.7496	42
43	"	"	"	20	"	34.66	12.8100	-0.1737	"	-0.4259	-0.1849	11.7419	43
44	"	"	"	"	"	36.16	12.7817	"	"	-0.4026	"	11.7369	44
45	"	Invar	"	12	"	36.45	13.0125	+1.6874	"	-0.0215	-2.2030	12.1918	45
46	"	"	"	"	"	37.21	12.9383	"	"	-0.0209	"	12.1182	46
47	"	"	"	20	"	32.40	12.5983	+0.6074	"	-0.0250	-0.7638	12.1333	47
48	"	"	"	"	"	38.23	12.4167	"	"	-0.0201	"	11.9566	48
49	"	"	"	30	"	38.57	12.0792	+0.2700	"	-0.0198	-0.1608	11.8850	49
50	"	"	"	"	"	39.36	12.0600	"	"	-0.0191	"	11.8665	50
51	"	Steel	"	"	"	39.69	12.5237	-0.0772	"	-0.3476	-0.0105	11.8048	51
52	"	"	"	"	"	39.41	12.5240	"	"	-0.3519	"	11.8008	52

PLATE XIV Cont.

Observation No.	Length of Tape ft.	Kind of Tape	Dist between Supports	Pull lbs.	Grade Line	Average Temperature °Fahr.	Measured Distance ft	Sag Correction ft	Grade Correction ft.	Temperature Correction ft.	Length Correction ft.	Corrected Distance ft.	Observation No.
53	100	Steel	50	12	A	41.08	2412.9958	-0.2934	-0.4100	-0.3127	-0.1545	2411.8252	53
54	"	"	"	"	"	42.60	12.8117	"	"	-0.2900	"	11.6638	54
55	"	"	"	20	"	41.30	12.6225	-0.1057	"	-0.3095	+0.0363	11.8336	55
56	"	"	"	"	"	41.34	12.6100	"	"	-0.3089	"	11.8217	56
57	"	"	"	30	"	41.68	12.3375	-0.0470	"	-0.3039	+0.2628	11.8394	57
58	"	"	"	"	"	40.63	12.3642	"	"	-0.3195	"	11.8505	58
59	"	Invar	"	12	"	39.64	13.0150	+1.6874	"	-0.0215	-2.3879	11.8830	59
60	"	"	"	"	"	38.57	13.0483	"	"	-0.0225	"	11.9153	60
61	"	"	"	20	"	66.90	12.5108	+0.6074	"	+0.0047	-0.7236	11.9893	61
62	"	"	"	"	"	64.88	12.5042	"	"	+0.0028	"	11.9808	62
63	"	"	"	30	"	62.74	12.1775	+0.2700	"	+0.0007	-0.0965	11.9417	63
64	"	"	"	"	"	60.43	12.1716	"	"	-0.0015	"	11.9336	64
65	"	"	"	12	"	58.72	13.0491	+1.6874	"	-0.0031	-2.3879	11.9355	65
66	"	Steel	100	30	"	49.27	12.4466	-0.1877	"	-0.1903	+0.2629	11.9215	66
67	"	"	"	"	"	52.11	12.3667	"	"	-0.1479	"	11.8840	67
68	"	"	"	20	"	54.07	12.7283	-0.4225	"	-0.1185	+0.0362	11.8135	68
69	"	"	"	"	"	55.92	12.7033	"	"	-0.0909	"	11.8161	69
70	"	Invar	"	30	"	58.18	12.4366	0.00	"	-0.0037	-0.0965	11.9254	70
71	"	"	"	"	"	60.00	12.4291	"	"	-0.0019	"	11.9207	71
72	"	"	"	20	"	61.05	13.0708	"	"	-0.0009	-0.7236	11.9363	72
73	"	"	"	"	"	62.45	13.0725	"	"	+0.0004	"	11.9393	73
74	300	"	75	12	C	70.41	13.5600	+0.9841	-0.2935	+0.0071	-2.2030	12.0546	74
75	"	"	"	"	"	71.10	13.5083	"	"	+0.0077	"	12.0035	75
76	"	"	"	20	"	71.33	12.6383	+0.3544	"	+0.0079	-0.7638	11.9432	76
77	"	"	"	"	"	71.27	12.7741	"	"	+0.0078	"	12.0789	77
78	"	"	"	30	B	71.66	12.1383	+0.1575	-0.2836	+0.0081	-0.1608	11.8595	78

PLATE XIV Cont.

Observation No.	Length of Tape ft	Kind of Tape	Dist between Supports	Pull lbs.	Grade Line	Average Temperature °Fahr.	Measured Distance ft.	Sag Correction ft.	Grade Correction ft.	Temperature Correction ft.	Length Corrections ft.	Corrected Distance ft.	Observation No.
79	300	Invar	75	30	B	71.80	2412.1442	+0.1575	-0.2836	+0.0083	-0.1608	2411.8656	79
80	"	Steel	"	12	C	71.33	13.3308	-1.0856	-0.2935	+0.1454	-0.3296	11.7674	80
81	"	"	"	"	"	71.47	13.4000	"	"	+0.1475	"	11.8387	81
82	"	"	"	20	"	68.17	12.7225	-0.3907	"	+0.0961	-0.1849	11.9494	82
83	"	"	"	"	"	68.07	12.7142	"	"	+0.0946	"	11.9396	83
84	"	"	"	30	B	67.60	12.2858	-0.1736	-0.2836	+0.0872	-0.0105	11.9053	84
85	"	"	"	"	"	66.33	12.2950	"	"	+0.0675	"	11.8948	85
86	"	"	25	12	"	66.85	12.5291	-0.1207	"	+0.0756	-0.3296	11.8708	86
87	"	"	"	20	"	66.37	12.3175	-0.0434	"	+0.0681	-0.1849	11.8737	87
88	"	"	100	"	D	76.37	12.7933	-0.6946	-0.3436	+0.2239	"	11.7941	88
89	"	"	"	"	"	78.23	12.7134	"	"	+0.2529	"	11.7432	89
90	"	"	"	30	B	78.16	12.1283	-0.3087	-0.2836	+0.2518	-0.0105	11.7173	90
91	"	"	"	"	"	79.47	12.0954	"	"	+0.2722	"	11.7048	91
92	"	Invar	"	20	D	79.60	12.9825	0.00	-0.3436	+0.0149	-0.7638	11.8900	92
93	"	"	"	"	"	80.81	13.0216	"	"	+0.0159	"	11.9301	93
94	"	"	"	30	B	80.28	12.3616	"	-0.2836	+0.0154	-0.1608	11.8726	94
95	"	"	"	"	"	80.96	12.3917	"	"	+0.0160	"	11.9033	95
96	"	Steel	"	20	D	51.77	13.2950	-0.6946	-0.3436	-0.1594	-0.1849	11.9125	96
97	"	"	75	12	C	50.73	13.7642	-1.0856	-0.2935	-0.1756	-0.3296	11.8799	97
98	"	"	25	30	B	51.10	12.4433	-0.0193	-0.2836	-0.1698	-0.0105	11.9601	98
99	"	"	"	20	"	50.46	12.6208	-0.0434	"	-0.1798	-0.1849	11.9291	99
100	"	Invar	75	"	C	48.66	12.6550	+0.3544	-0.2935	-0.0113	-0.7638	11.9408	100
101	"	"	50	"	B	48.53	12.4317	+0.6074	-0.2836	-0.0114	"	11.9803	101
102	"	"	25	"	B	48.80	12.2933	+0.7592	"	-0.0111	"	11.9940	102
103	100	Steel	50	12	A	48.74	12.9500	-0.2934	-0.4100	-0.1982	-0.1544	11.8940	103
104	"	Invar	25	20	"	46.29	12.3683	+0.7592	"	-0.0151	-0.7236	11.9788	104
105	"	"	"	30	"	45.78	12.1358	+0.3375	"	-0.0156	-0.0965	11.9512	105

PLATE XV.

Grouping of Measurements With Reference to Distance Between Supports

Number	Tape	Distance Between Supports Ft.	Pull Lb.	Measured Length Ft.	Corrected Length Ft.	
13	100 S	25	12	2412.5842	2411.7183	
14				2412.6833	2411.8169	
33				2412.8717	2411.8391	
36				2412.8633	2411.8386	
				Average	2411.8107	
15	100 S	25	20	2412.2900	2411.7114	
16				2412.3750	2411.8284	
34				2412.6883	2411.8362	
				Average	2411.7920	
17	100 S	25	30	2411.9983	2411.7148	
18				2412.1033	2411.8208	
35				2412.4333	2411.8616	
				Average	2411.7991	
19	100 I	25	12	2412.6950	2411.9314	
20				2412.6517	2411.9567	
37				2412.6617	2411.9476	
40				2412.6767	2411.9606	
				Average	2411.9491	
21	100 I	25	20	2412.3700	2411.9863	
22				2412.4198	2412.0366	
38				2412.4042	2412.0039	
104				2412.3883	2411.9798	N
				Average	2412.0023	
23	100 I	25	30	2412.0942	2411.9153	
24				2412.2342	2412.0540	
39				2412.1967	2411.9304	
105				2412.1750	2411.9512	N
				Average	2411.9627	
2	300 S	25	12	2412.7417	2411.7866	
25				2413.1783	2411.8799	
28				2412.9592	2411.7945	
86				2412.5291	2411.8708	N
				Average	2411.8330	

PLATE XV. (continued)

Number	Tape	Distance Between Supports Ft.	Pull Lb.	Measured Length Ft.	Corrected Length Ft.	
3	300 S	25	20	2412.4933	2411.7816	
4				2412.4033	2411.7601	
30				2412.8067	2411.9307	
87				2412.3175	2411.8737	L
99				2412.3708	2411.9291	N
				Average	2411.8350	
11	300 S	25	30	2412.5783	2412.0681	
12				2412.3450	2411.8412	
27				2412.7900	2412.0203	
98				2412.4433	2411.9001	N
				Average	2411.9324	
5	300 I	25	12	2412.5377	2412.1436	
6				2412.5343	2412.1435	
29				2412.5500	2412.1512	
32				2412.5000	2412.1607	
				Average	2412.1512	
7	300 I	25	20	2412.2045	2411.9880	
8				2412.2005	2411.9854	
30				2412.2750	2411.9831	
103				2412.2933	2411.9940	N
				Average	2411.9904	
9	300 I	25	30	2412.0067	2411.8914	
10				2412.0083	2411.8923	
31				2412.0117	2411.8889	
				Average	2411.8909	
53	100 S	50	12	2412.8958	2411.8252	
54				2412.8117	2411.8038	
103				2412.9500	2411.8940	N
				Average	2411.7943	
55	100 S	50	20	2412.6225	2411.8535	
56				2412.6100	2411.8217	
				Average	2411.8376	
57	100 S	50	30	2412.3375	2411.8394	
58				2412.3642	2411.8505	
				Average	2411.8419	
59	100 I	50	12	2413.0150	2411.8830	
65				2413.0401	2411.9355	
60				2413.0483	2411.9153	
				Average	2411.9113	

PLATE XV. (continued)

Number	Tape	Distance Between Supports Ft.	Pull Lb.	Measured Length Ft.	Corrected Length Ft.	
61	100 I	50	20	2412.5108	2411.9893	
62				2412.5042	2411.9808	
				Average	2411.9850	
63	100 I	50	30	2412.1775	2411.9417	
64				2412.1716	2411.9336	
				Average	2411.9376	
41	300 S	50	12	2413.2500	2411.7167	
42				2413.2750	2411.7496	
				Average	2411.7331	
43	300 S	50	20	2412.8100	2411.7419	
44				2412.7817	2411.7369	
				Average	2411.7394	
51	300 S	50	30	2412.5237	2411.8048	
52				2412.5240	2411.8008	
				Average	2411.8028	
45	300 I	50	12	2413.0125	2412.1918	
46				2412.9383	2412.1182	
				Average	2412.1550	
47	300 I	50	20	2412.5983	2412.1333	
48				2412.4167	2411.9566	
101				2412.4317	2411.9803	M
				Average	2412.0234	
49	300 I	50	30	2412.0792	2411.8850	
50				2412.0600	2411.8665	
				Average	2411.8757	
80	300 S	75	12	2413.3308	2411.7674	
81				2413.4000	2411.8387	
97				2413.7642	2411.8799	M
				Average	2411.8287	
82	300 S	75	20	2412.7225	2411.9494	M
83				2412.7142	2411.9396	M
				Average	2411.9445	
84	300 S	75	30	2412.2858	2411.9053	M
85				2412.2850	2411.8948	M
				Average	2411.9000	
74	300 I	75	12	2413.5600	2412.0548	
75				2413.5083	2412.0035	
				Average	2412.0290	

PLATE XV. (continued)

Number	Tape	Distance Between Supports Ft.	Pull Lb.	Measured Length Ft.	Corrected Length Ft.
76	300 I	75	20	2412.6383	2411.9432
77				2412.7741	2412.0789
100				2412.6550	2411.9408 N
				Average	2411.9876
78	300 I	75	30	2412.1383	2411.9595
79				2412.1442	2411.8656
				Average	2411.8697
68	100 S	100	20	2412.7283	2411.8135
69				2412.7033	2411.8161
				Average	2411.8148
66	100 S	100	30	2412.4466	2411.9215
67				2412.3667	2411.8840
				Average	2411.9027
72	100 I	100	20	2413.0708	2411.9363
73				2413.0725	2411.9393
				Average	2411.9378
70	100 I	100	30	2412.4366	2411.9254
71				2412.4291	2411.9207
				Average	2411.9230
88	300 S	100	20	2412.7933	2411.7941
89				2412.7134	2411.7432
96				2413.2950	2411.9125 N
				Average	2411.8166
90	300 S	100	30	2412.1283	2411.7173
91				2412.0954	2411.7048
				Average	2411.7110
92	300 I	100	20	2412.9825	2411.8900
93				2413.0216	2411.9301
				Average	2411.9100
94	300 I	100	30	2412.3616	2411.8726
95				2412.3917	2411.9033
				Average	2411.8879

Note: Measurements followed by N are night measurements.

PLATE XVI.

Grouping of Measurements With Reference to Kind of Tape

Number	Tape	Distance Between Culverts Ft.	Pull Lb.	Measured Length Ft.	Corrected Length Ft.	Average Number
3	300 S	25	12	2412.7417	2411.7808	
25				2413.1782	2411.8789	
28				2412.9592	2411.7945	
30				2412.5291	2411.3705	
				Average	2411.8337	1
3	300 S	25	20	2412.4923	2411.7918	
4				2412.4633	2411.7601	
26				2412.8067	2411.8307	
27				2412.5175	2411.5357	
29				2412.8008	2411.8201	
				Average	2411.8276	2
11	300 S	25	30	2412.5703	2412.0681	
12				2412.3450	2411.8412	
27				2412.9700	2412.0203	
28				2412.4435	2411.9001	
				Average	2411.9324	3
41	300 S	50	12	2413.2600	2411.7167	
42				2413.2750	2411.7496	
				Average	2411.7331	4
43	300 S	50	20	2412.8100	2411.7419	
44				2412.7817	2411.7139	
				Average	2411.7394	5
51	300 S	50	30	2412.5237	2411.8048	
52				2412.5240	2411.8008	
				Average	2411.8028	6
80	300 S	75	12	2413.5508	2411.7074	
81				2413.4000	2411.8367	
97				2413.7642	2411.8199	
				Average	2411.8267	7
82	300 S	75	20	2412.7235	2411.8494	
83				2412.7142	2411.8396	
				Average	2411.8445	8
84	300 S	75	30	2412.8858	2411.8053	
85				2412.8950	2411.8148	
				Average	2411.8000	9

PLATE XVI. (continued)

Number	Tape	Distance Between Supports Ft.	Pull Lb.	Measured Length Ft.	Corrected Length Ft.	Average Number
88	300 S	100	20	2413.7933	2411.7941	
89				2413.7134	2411.7432	
96				2413.8050	2411.8160	
				Average	2411.8160	10
90	300 S	100	30	2413.1983	2411.7173	
91				2413.0954	2411.7046	
				Average	2411.7110	11
5	300 I	25	12	2413.6277	2413.1438	
6				2413.6343	2413.1498	
29				2413.6500	2413.1518	
32				2413.6600	2413.1607	
				Average	2413.1519	12
7	300 I	25	20	2413.2048	2411.8000	
8				2413.2905	2411.8034	
30				2413.2750	2411.8881	
102				2413.2933	2411.8940N	
				Average	2411.8804	13
9	300 I	25	30	2413.0067	2411.8914	
10				2413.0083	2411.8933	
31				2413.0117	2411.8809	
				Average	2411.8869	14
45	300 I	50	12	2413.0195	2413.1918	
46				2413.0383	2413.1107	
				Average	2413.1550	15
47	300 I	50	20	2413.5983	2413.1332	
48				2413.4167	2411.8506	
101				2413.4517	2411.8857	
				Average	2413.0334	16
49	300 I	50	30	2413.0783	2411.8830	
50				2413.0600	2411.8683	
				Average	2411.8757	17
74	300 I	75	12	2413.5600	2413.0840	
75				2413.5083	2413.0035	
				Average	2413.0390	18
76	300 I	75	20	2413.6783	2411.9452	
77				2413.7741	2413.0789	
100				2413.6550	2411.9408N	
				Average	2411.9876	19

PLATE XVI. (continued)

Number	Tape	Distance Between Supports Ft.	Pull Lb.	Measured Length Ft.	Corrected Length Ft.	Average Number
78	300 I	75	30	2412.1383	2411.8595	
79				2412.1442	2411.8656	
				Average	2411.8627	20
92	300 I	100	20	2412.9825	2411.9900	
93				2413.0716	2411.9801	
				Average	2411.9100	21
94	300 I	100	30	2412.8618	2411.8726	
95				2412.8817	2411.9033	
				Average	2411.8879	22
13	100 S	25	12	2412.5842	2411.7183	
14				2412.6833	2411.8468	
33				2412.8717	2411.8391	
36				2412.8633	2411.8306	
				Average	2411.8107	23
15	100 S	25	20	2412.6900	2411.7114	
16				2412.8750	2411.8984	
34				2412.8283	2411.8502	
				Average	2411.7920	24
17	100 S	25	30	2411.9992	2411.7148	
18				2412.1033	2411.8200	
35				2412.4333	2411.8610	
				Average	2411.7991	25
53	100 S	50	12	2412.8958	2411.8052	
54				2412.9117	2411.8038	
103				2412.9500	2411.8040	
				Average	2411.7943	26
55	100 S	50	20	2412.8825	2411.8336	
56				2412.8100	2411.8217	
				Average	2411.8276	27
57	100 S	50	30	2412.8375	2411.8394	
58				2412.8642	2411.8505	
				Average	2411.8449	28
68	100 S	100	20	2412.7783	2411.8135	
69				2412.7983	2411.8161	
				Average	2411.8148	29
66	100 S	100	30	2412.4466	2411.8215	
67				2412.3667	2411.8040	
				Average	2411.8027	30

PLATE XVI. (continued)

Number	Tape	Distance Between Supports Ft.	Pull Lb.	Measured Length Ft.	Corrected Length Ft.	Average Number
19	100 I	25	12	2412.6250	2411.9314	
20				2412.6517	2411.9567	
37				2412.6617	2411.9476	
40				2412.6767	2411.9606	
				Average	2411.9491	31
21	100 I	25	20	2412.3700	2411.9863	
22				2412.4192	2412.0306	
38				2412.4042	2412.0039	
104				2412.3683	2411.9788N	
				Average	2412.0023	32
23	100 I	25	30	2412.0942	2411.9153	
24				2412.0342	2412.0540	
39				2412.1267	2411.9304	
105				2412.1358	2411.9512N	
				Average	2411.9627	33
59	100 I	50	12	2413.0150	2411.9830	
65				2413.0491	2411.9355	
66				2413.0485	2411.9153	
				Average	2411.9113	34
61	100 I	50	20	2412.5108	2411.9895	
62				2412.5042	2411.9808	
				Average	2411.9850	35
63	100 I	50	30	2412.1775	2411.9417	
64				2412.1716	2411.9336	
				Average	2411.9376	36
72	100 I	100	20	2413.0708	2411.9363	
73				2413.0725	2411.9393	
				Average	2411.9378	37
70	100 I	100	30	2412.4566	2411.9254	
71				2412.4201	2411.9207	
				Average	2411.9230	38

Note: Measurements followed by N are night measurements.

PROBABLE ERRORS

The probable errors tabulated on page 48 were computed for each tape from the averages of measurements (see Plate XVI, page 39.) taken with the same pull and the same distance between supports. For the purpose of comparison, it would have been more desirable to obtain a probable error for the individual observation, so that measurements taken under the same conditions but with different tapes could be compared; as for example the measurements in groups 1 and 12, Plate XVI, page 39. Since there are only two measurements in some groups, this plan had to be abandoned because a probable error cannot be computed for one of two results. Therefore, the probable error of the average of the separate groups was computed instead.

The formula for the probable error of the average is $R = 0.6745\left(\frac{\Sigma V^2}{n-1}\right)^{\frac{1}{2}}$, where R is the probable error of the average referred to before; V is the difference of any average from the mean of the averages; and n is the number of averages. The probable error of the mean of the averages is determined from the formula $R_o = \frac{R}{\sqrt{n}}$, where R_o is the probable error of the mean of the averages.

In Plates XVII to XX, pages 44to 47,the computations are shown for the determination of the probable error of the averages mentioned above, and for the mean of the averages, using the formulas just mentioned. The numbers in the column with the heading "Value", are the averages of the corrected lengths in the respective groupings indicated by the average number. The tabulated form for the computations here again simplified the work as may be readily seen. At the bottom of each Plate is the prob-

PLATE XVII.

PROBABLE ERROR

100-Foot Steel

Average No.	value	v	v^2
23	11.8107	0.0126	0.00015876
24	11.7920	0.0313	0.00097969
25	11.7991	0.0242	0.00058564
26	11.7943	0.0290	0.00084100
27	11.8276	0.0043	0.00001849
28	11.8449	0.0216	0.00046656
29	11.8148	0.0085	0.00007225
30	11.9027	0.0794	0.00630436
Ave.	11.8233		0.00942675 = Σv^2

$$\sqrt{\frac{\Sigma v^2}{n-1}} = \sqrt{\frac{0.0094}{7}} = 0.0367$$

$$R = 0.0247$$

$$R_0 = \frac{R}{\sqrt{n}} = 0.00874$$

PLATE XVIII.

PROBABLE ERROR

100-Foot Invar

Average No.	Value	V	V^2
31	11.9491	0.0020	0.00000400
32	12.0023	0.0512	0.00262144
33	11.9627	0.0116	0.00013456
34	11.9113	0.0398	0.00158404
35	11.9850	0.0339	0.00114921
36	11.9376	0.0135	0.00018225
37	11.9378	0.0133	0.00017689
38	11.9230	0.0281	0.00078961
Ave.	11.9511		0.0066420($= \Sigma V^2$

$$\sqrt{\frac{\Sigma V^2}{n-1}} = \sqrt{\frac{0.00664}{7}} = 0.0308$$

$$R = 0.0208$$

$$R_o = \frac{R}{\sqrt{n}} = 0.00736$$

PLATE XIX.

PROBABLE ERROR

300-Foot Steel

Average No.	Value	V	V^2
1	11.8330	0.0042	0.00001764
2	11.8350	0.0062	0.00003844
3	11.9724	0.1436	0.02062096
4	11.7331	0.0957	0.00915849
5	11.7394	0.0894	0.00799236
6	11.8028	0.0260	0.00067600
7	11.8287	0.0001	0.00000001
8	11.9445	0.1157	0.01338649
9	11.9000	0.0712	0.00506944
10	11.8166	0.0122	0.00014884
11	11.7110	0.1178	0.01387684
Ave.	11.8288		0.07098551= ΣV^2

$$\sqrt{\frac{\Sigma V^2}{n-1}} = \sqrt{\frac{0.07098}{10}} = 0.08425$$

$$R = 0.6745\sqrt{\frac{\Sigma V^2}{n-1}} = 0.0568$$

$$R_o = \frac{R}{\sqrt{n}} = 0.0171$$

PLATE XX.

PROBABLE ERROR

300-Foot Invar

Average No.	Value	V	V^2
12	12.1512	0.1738	0.03020644
13	11.9804	0.0030	0.00000900
14	11.8882	0.0892	0.00795664
15	12.1550	0.1776	0.03154176
16	12.0234	0.0460	0.00211600
17	11.8757	0.1017	0.01034289
18	12.0290	0.0516	0.00266256
19	11.9896	0.0102	0.00010404
20	11.8627	0.1147	0.01315609
21	11.9100	0.0674	0.00454276
22	11.8879	0.0895	0.00801025
Ave.	11.9774		0.11064845 $=\Sigma V^2$

$$\sqrt{\frac{\Sigma V^2}{n-1}} = \sqrt{\frac{0.1106}{10}} = 0.10519$$

$$R = 0.0709$$

$$R_o = \frac{R}{\sqrt{n}} = 0.0214$$

PLATE XXI.

TABULATED PROBABLE ERRORS

Tape	Average		Mean of Averages	
	Prob. Error	Equivalent to 1 in	Prob. Error	Equivalent to 1 in
300 ft. S	0.0568	42 400	0.01710	140 800
300 ft. I	0.0709	34 000	0.02140	112 500
100 ft. S	0.0247	97 500	0.00874	275 500
100 ft. I	0.0208	115 800	0.00736	327 000

able error, R, for the averages in that particular group, and also the value of R_o for the mean of the averages.

As may readily be seen from the table on page 48, the probable error of the mean of the averages for the 100-ft. invar measurements is considerably less than that for the 100-ft. steel, but the probable error for the 300-ft. invar measurements is somewhat greater than that for the 300-ft. steel. The former of these comparisons conforms with expectations before the measurements were taken, and it was expected that the comparison for the 300-ft. tapes would be the same. The basis for the expectations was that the temperature correction was the most unreliable and variable factor to be considered, since the exact temperature of the tape is not likely to be indicated by thermometer readings.

It is roughly estimated, but with no substantial proof, that when the sun was shining, the temperature of the ground and the tape was about 10 degrees higher than that registered by the thermometers which hung from hooks about 2 ft. above the ground.

The grade correction is in most cases a constant for both steel and invar tapes, and the length correction is without doubt correct, as the tapes were standardized at Washington, D.C., before any measurements were taken. The correction for sag is also eliminated as a controlling factor in the final length of the base-line, as it can be readily computed by formula as shown on page 36 although there is some doubt in regard to the exact value of the pull at all points along the tape, as explained later in the conclusions.

With the temperature correction, then, as the controlling factor in the final length of the base-line, it seems quite evident that the invar tape measurements should be the more exact,

since the coefficient of expansion of invar is about 1/20 of that of steel.

The reason for the probable error of the measurements taken with the 300-ft. invar tape exceeding the probable error of the measurements taken with the 300-ft. steel tape, is indeed unknown, and would probably justify further investigation.

CONCLUSIONS

The conslusions which have been deduced from this investigation of measurements with steel and invar tapes are as follows:

1. A consistent variation of results with different pulls and different distances between supports was expected, but the results obtained are unaccountably inconsistent, and very few definite conclusions can be drawn.

2. A comparison of the measurements tabulated in PlateXV, page 35, indicates that the results obtained with a 12 lb. pull are not as consistent as those obtained with a greater pull. This is evidently due to the friction of the tape on the supports affecting the tension considerably towards the middle of the tape, sometimes increasing it and at other times decreasing it. With a 12 lb. pull on the 300-ft. tape, the tension registered by the balance at one end of the tape could be varied by about 2 lbs. without producing a noticeable effect upon the balance at the other end.

3. A comparison of results further shows that measurements made with the support stakes 25 and 50 ft. apart are more consistent than those made with the supports 75 and 100 ft. apart. For instance in Plate XVI, page 39, group No. 14, measurements taken with the supports 25 ft. apart, check within 0.01 ft., and in the

same Plate, group No. 17, measurements taken with the supports 50 ft. apart, check within 0.02 ft., while in group No. 18 the measurements taken with supports at 75 ft. intervals, check within about 0.05 ft., and in group No. 21 with 100-ft. intervals between supports the measurements check within 0.04 ft. With a pull of 12 lbs., measurements with supports more than 75 feet apart were practically worthless.

4. Results obtained from night measurements are considerably more consistent than those obtained from day measurements. This fact is more marked in the results obtained with the steel tapes than in those obtained with the invar tapes. Plate XVI, page 39, group No. 8, night measurements with the steel tape show a discrepancy of 0.01 ft., while in group No. 7 of the same Plate, the measurements taken with a steel tape during the day show a discrepancy of 0.11 ft. An examination of the above mentioned Plate, shows that measurements taken with invar tapes during the day check about as closely as those taken during the night. The inconsistency of day measurements with steel tapes was doubtless due to the impossibility of obtaining the exact temperature of the tape at all times.

5. The average rate of progress in the measurement of the base-line with the 300-ft. tapes was 16 minutes per measurement, which is equivalent to 1.71 miles per hour; while the average with the 100-ft. tapes was 21 minutes per measurement, which is equivalent to 1.30 miles per hour. It was also found that with the use of electric flash lights, the speed of night measurements approximated that of day measurements.

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